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Exercises for chapter 4 (more in course) - solutions

1. Let $\mathcal{H}_0$ be $\mathcal{N}(0,1)$. We have $n = 1$ to keep the maths easy. Choose $\alpha = 0.05$ and test against

a) $\mathcal{H}_1$ being $\mathcal{N}(1,1)$ and compute the β too.

1. $\mathcal{H}_0 : \mu = 0$ vs $\mathcal{H}_1 : \mu = 1$
2. $\alpha = 0.05$ and $n = 1$.
3. choose (good) test statistic: x_1
4. distribution of X_1 under $\mathcal{H}_0$: $\mathcal{N}(0,1)$
5. critical value: 1.64, so if $x_1 < 1.64$, we say $\mathcal{H}_0$, if $x_1 \geq 1.64$, we say $\mathcal{H}_1$.

As choosen, $\alpha = 5\%$; Notation from now on: $\mathcal{N}(1,1) \leq 1.64$ means $X \leq 1.64$ where X is $\mathcal{N}(1,1)$. $\beta = P[\mathcal{N}(1,1) \leq 1.64] = P[\mathcal{N}(0,1) \leq 0.64] = 0.7389$.

b) $\mathcal{H}_1$ being $\mathcal{N}(2,1)$ and compute the β too.

1. $\mathcal{H}_0 : \mu = 0$ vs $\mathcal{H}_1 : \mu = 2$
2. $\alpha = 0.05$ and $n = 1$.
3. choose (good) test statistic: x_1
4. distribution of X_1 under $\mathcal{H}_0$: $\mathcal{N}(0,1)$
5. critical value: 1.64, so if $x_1 < 1.64$, we say $\mathcal{H}_0$, if $x_1 \geq 1.64$, we say $\mathcal{H}_1$.

As choosen, $\alpha = 5\%$. $\beta = P[\mathcal{N}(2,1) \leq 1.64] = P[\mathcal{N}(0,1) \leq -0.36] = 0.3594$.

Please notice, that mutatis mutandis all is the same in points 1-5. From now on we only write the last point:

c) $\mathcal{H}_1$ being $\mathcal{N}(3,1)$ and compute the β too.

As choosen, $\alpha = 5\%$. $\beta = P[\mathcal{N}(3,1) \leq 1.64] = P[\mathcal{N}(0,1) \leq -1.36] = 0.0869$.

d) $\mathcal{H}_1$ being $\mathcal{N}(4,1)$ and compute the β too.
As choosen, $\alpha = 5\%$. $\beta = P[\mathcal{N}(4,1) \leq 1.64] = P[\mathcal{N}(0,1) \leq -2.36] = 0.0094$.

e) Summarize results from a)-d). Does it make sense?

Test is always the same, as it is constructed under $\mathcal{H}_0$. The critical point is 1.64. But as the two distributions are separated better and better, we see a smaller and smaller probability for type 2 error (so more power).

Obviously: 1.64 and 1.96 are very important numbers for statisticians!

2. Medical treatment: You have 51 patients, measure blood pressure before treatment and after treatment. Data at hand is difference: $x_1, \ldots, x_{51}$. Pharmaceutical company claims, blood pressure is lower with treatment than without. σ has been estimated to be 8.4; $\overline{x} = -2.3$. Make a statistical test. We will do this at $\alpha = 0.05$ and later $\alpha = 0.025$.

1. One sided, as pharmaceutical company claims to *reduce* blood pressure: $\mathcal{H}_0 : \mu = 0$ ($\mu \geq 0$ would be ok too) vs $\mathcal{H}_1 : \mu < 0$.
2. $\alpha = 0.05$ (and later $\alpha = 0.025$) and $n = 51$.
3. choose (good) test statistic: $t := \frac{\sqrt{51}\bar{x}}{\hat{\sigma}}$
4. distribution of T under $\mathcal{H}_0$: t_{50} (we are making a (pooled) t-test...)
5. critical values a are qt(0.025, 50)=-2.008559 and qt(0.05, 50)=-1.675905, so if $t < a$, we say $\mathcal{H}_1$, if $t \geq a$, we say $\mathcal{H}_0$.
6. sqrt(51)*(-2.3)/8.4= -1.955391 and that means we reject $\mathcal{H}_0$ if $\alpha = 0.05$ and stay with $\mathcal{H}_0$ if $\alpha = 0.025$.

3. Tablets are weighted. We got the following weight in grams:

$$1.19, 1.23, 1.18, 1.21, 1.27, 1.17, 1.15, 1.14.$$

a) Test, whether the average weight is 1.2 g (two-sided) at 5 %.
b) Test, whether the average weight is less than 1.2 g (one-sided), and not as claimed 1.2 g, at 5 %.

Give precisely the two hypothesis.

We solve both a) and b) in one, as most is common:

1. a) $\mathcal{H}_0 : \mu = 1.2$ vs $\mathcal{H}_1 : \mu \neq 1.2$ and in b) $\mathcal{H}_0 : \mu \geq 1.2$ vs $\mathcal{H}_1 : \mu < 1.2$
2. $\alpha = 0.05$ and $n = 8$.
3. choose (good) test statistic: $t := \frac{\sqrt{8}(\bar{x}-1.2)}{\hat{\sigma}}$
4. distribution of T under $\mathcal{H}_0$: t_7 (we are making a t-test...)
5. critical values are qt(0.025, 7)=-2.364624 and qt(0.05, 7)=-1.894579, so in a) if $t \in [-2.364624, 2.364624]$, we say $\mathcal{H}_0$, if $t \notin [-2.364624, 2.364624]$, we say $\mathcal{H}_1$. In b): if $t > -1.894579$, we say $\mathcal{H}_0$, if $t \leq -1.894579$, we say $\mathcal{H}_1$
6. We need the following numbers:

   mean(c(1.19, 1.23, 1.18, 1.21, 1.27, 1.17, 1.15, 1.14))=1.1925

   sd(c(1.19, 1.23, 1.18, 1.21, 1.27, 1.17, 1.15, 1.14))=0.04301163

$\frac{\sqrt{8}(\bar{x}-1.2)}{\hat{\sigma}}$. We stick with $\mathcal{H}_0$ everytime.