

This script is an extract, with gaps, from the book “Introduction to the Mathematical Treatment of the Natural Sciences I - Analysis” by Christoph Luchsinger and Hans Heiner Storrer, Birkhäuser Scripts. As a student you should buy the book as well and work your way through it completely during the course MAT 182. You are allowed to save this PDF and modify it as you like, for your own use during MAT 182. For further use outside of MAT 182, please contact the lecturer, Christoph Luchsinger of the University of Zürich, in advance. The copyright remains with Birkhäuser!

2. VECTOR ARITHMETIC WITH COORDINATES

(2.2) The Cartesian coordinate system

We choose an origin O in space and a rectangular (instead of “rectangular” this is also referred to as “Cartesian”, after R. DESCARTES (in Latin, CARTESIUS) (1596–1650)) coordinate system with an x -axis, y -axis and z -axis, which are arranged in the usual ordering:

(2.3) The coordinates of a vector

(2.4) Arithmetic operations with coordinates

In Chapter 1 we defined the arithmetic operations for vectors in a geometric way. How can the coordinates be used to perform these operations computationally? In the following table we assemble the answer.

Let the vectors

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

be given. Then

(1)
$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix},$$

(2)
$$\vec{a} - \vec{b} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix},$$

(3)
$$r\vec{a} = \begin{pmatrix} ra_1 \\ ra_2 \\ ra_3 \end{pmatrix},$$

(4)
$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3,$$

(5)
$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}.$$

In addition the following formulae hold:

(6) The length of a vector \vec{a} is given by

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

(7) The angle φ between the vectors \vec{a} and \vec{b} is given by

$$\cos \varphi = \frac{\vec{a}\vec{b}}{|\vec{a}| |\vec{b}|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}.$$

Notes / Clarifications:

(2.5) Proofs for a few of these rules

(2.6) Length of a vector in all dimensions; equation of a circle/sphere

en passant: equation of a circle/sphere

(2.7) A few examples

Distance, angle

Area of a triangle, parametric equation of a line

Equation of a plane

Finally: www.youtube.com/watch?v=gfWpNCFjgfY

Important:

1. Next, read the corresponding chapter of the book yourself.
2. Solve at least 5 of the end-of-chapter exercises and compare your answers with those in the back of the book. If needed, solve more than 5.
3. Now you are ready to attend an exercise session. Print the relevant part of the exercise script *in advance*, read through the exercises there *in advance* and start to think about them yourself (for example, how you might approach solving them).
4. Next, solve the problems on the worksheet. Always try them first yourself. If it doesn't work, try with a tip from a fellow student. If it still doesn't work, look at another student's solution, wait 1 hour and try to solve it again from your own head. If none of that works: follow another student's solution (but be sure you understand it - in particular see that you aren't copying someone else's mistakes!)
5. Solve the corresponding problems from past exams in the course archive.